Example: Context-Free Pumping Lemma$_{JP}$

Use the JFLAP Context-Free Pumping Lemma game for the lemma

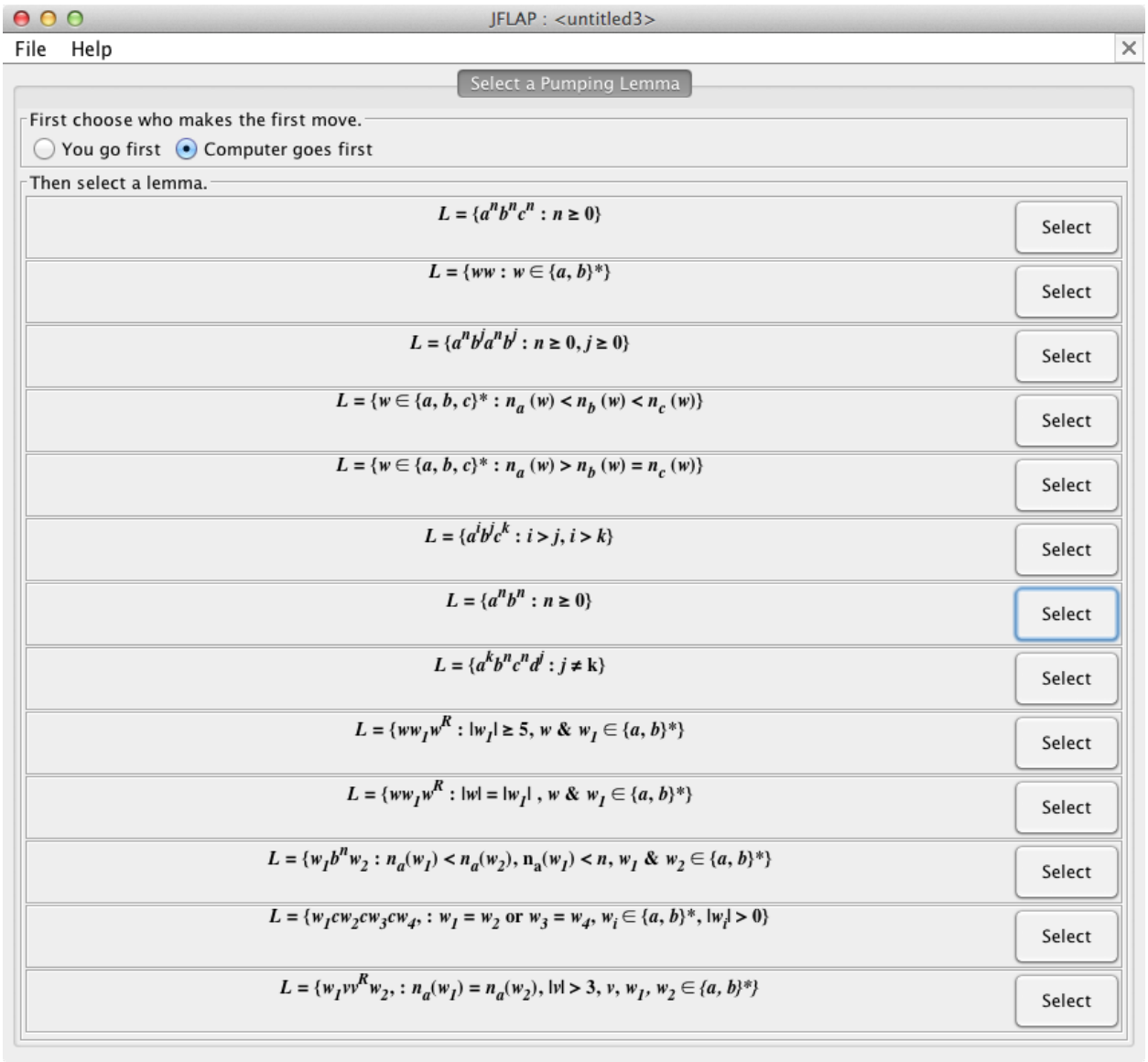
$$L = \{ a^n b^n : n \geq 0 \}$$

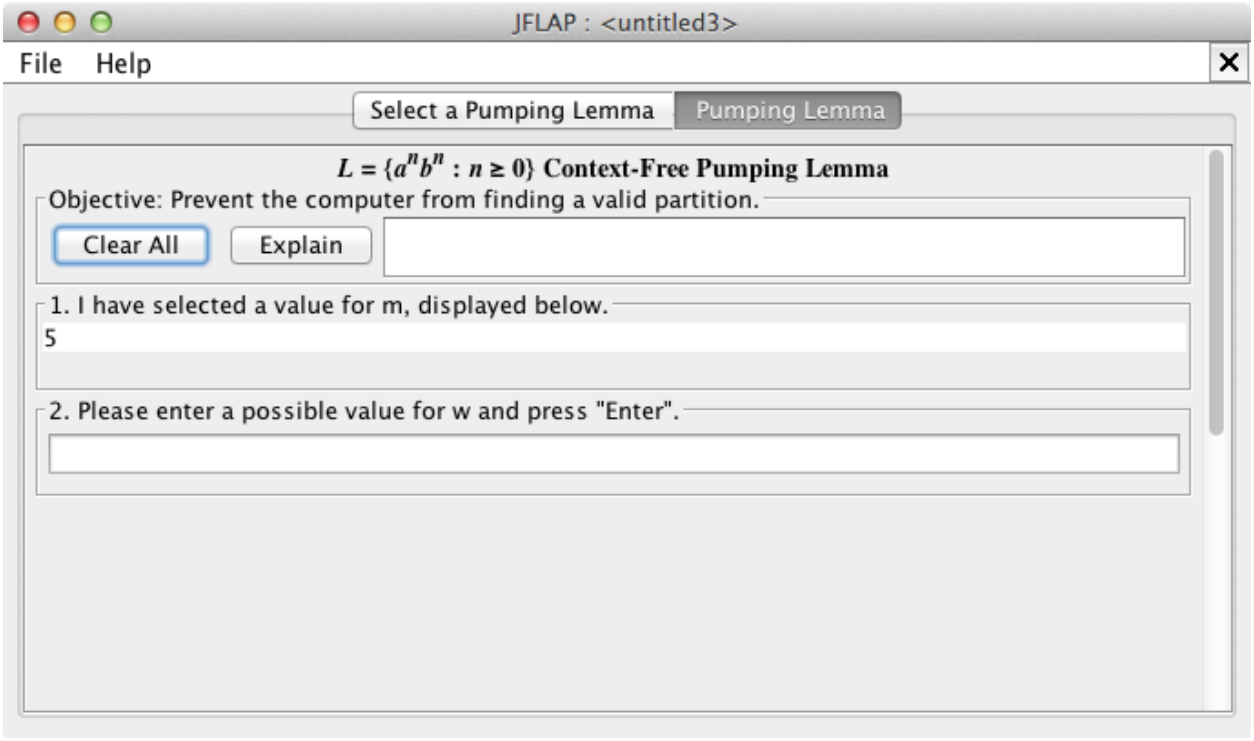
Recall that if L is a context-free language then there exists an integer m > 0 such that any w □ L with $|w| \geq m$ can be decomposed as the concatenation $w = uvxyz$, with $|vxy| \leq m$, $|vy| \geq 1$, and uv^ixy^iz □ L for all $i \geq 0$.

Consider what characteristics of a string are necessary for successful decomposition into five concatenated components, the second and fourth of which can be eliminated or repeated.

Walkthrough (see `CFPL_anbn.jff`)

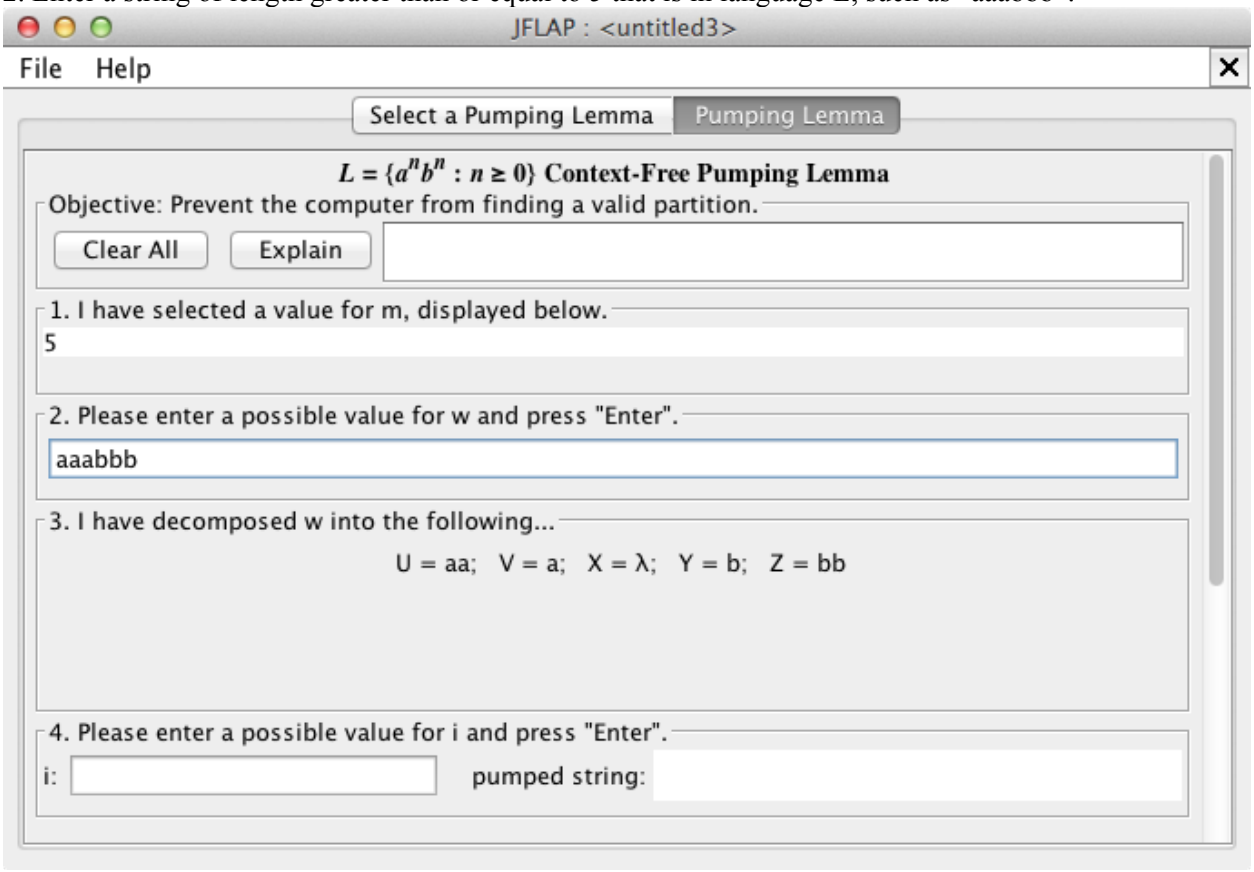
1. Enter the *Context-Free Pumping Lemma* game, select "Computer goes first", and choose "Select" to the right of "$L = \{ a^n b^n : n \geq 0 \}$".


JFLAP : <untitled3>
File
Help
Select a Pumping Lemma
First choose who makes the first move.
You go first
Computer goes first
Then select a lemma.
$L = \{a^n b^n c^n : n \geq 0\}$
Select
$L = \{ww : w \in \{a, b\}^*\}$
Select
$L = \{a^n b^j a^n b^j : n \geq 0, j \geq 0\}$
Select
$L = \{w \in \{a, b, c\}^* : n_a(w) < n_b(w) < n_c(w)\}$
Select
$L = \{w \in \{a, b, c\}^* : n_a(w) > n_b(w) = n_c(w)\}$
Select
$L = \{a^i b^j c^k : i > j, i > k\}$
Select
$L = \{a^n b^n : n \geq 0\}$
Select
$L = \{a^k b^n c^n d^j : j \neq k\}$
Select
$L = \{ww_1 w^R : |w_1| \geq 5, w \& w_1 \in \{a, b\}^*\}$
Select
$L = \{ww_1 w^R : |w| = |w_1|, w \& w_1 \in \{a, b\}^*\}$
Select
$L = \{w_1 b^n w_2 : n_a(w_1) < n_a(w_2), n_a(w_1) < n, w_1 \& w_2 \in \{a, b\}^*\}$
Select
$L = \{w_1 c w_2 c w_3 c w_4, : w_1 = w_2 \text{ or } w_3 = w_4, w_i \in \{a, b\}^*, |w_i| > 0\}$
Select
$L = \{w_1 v v^R w_2, : n_a(w_1) = n_a(w_2), |v| > 3, v, w_1, w_2 \in \{a, b\}^*\}$
Select



In this example, the Computer has selected m = 5.

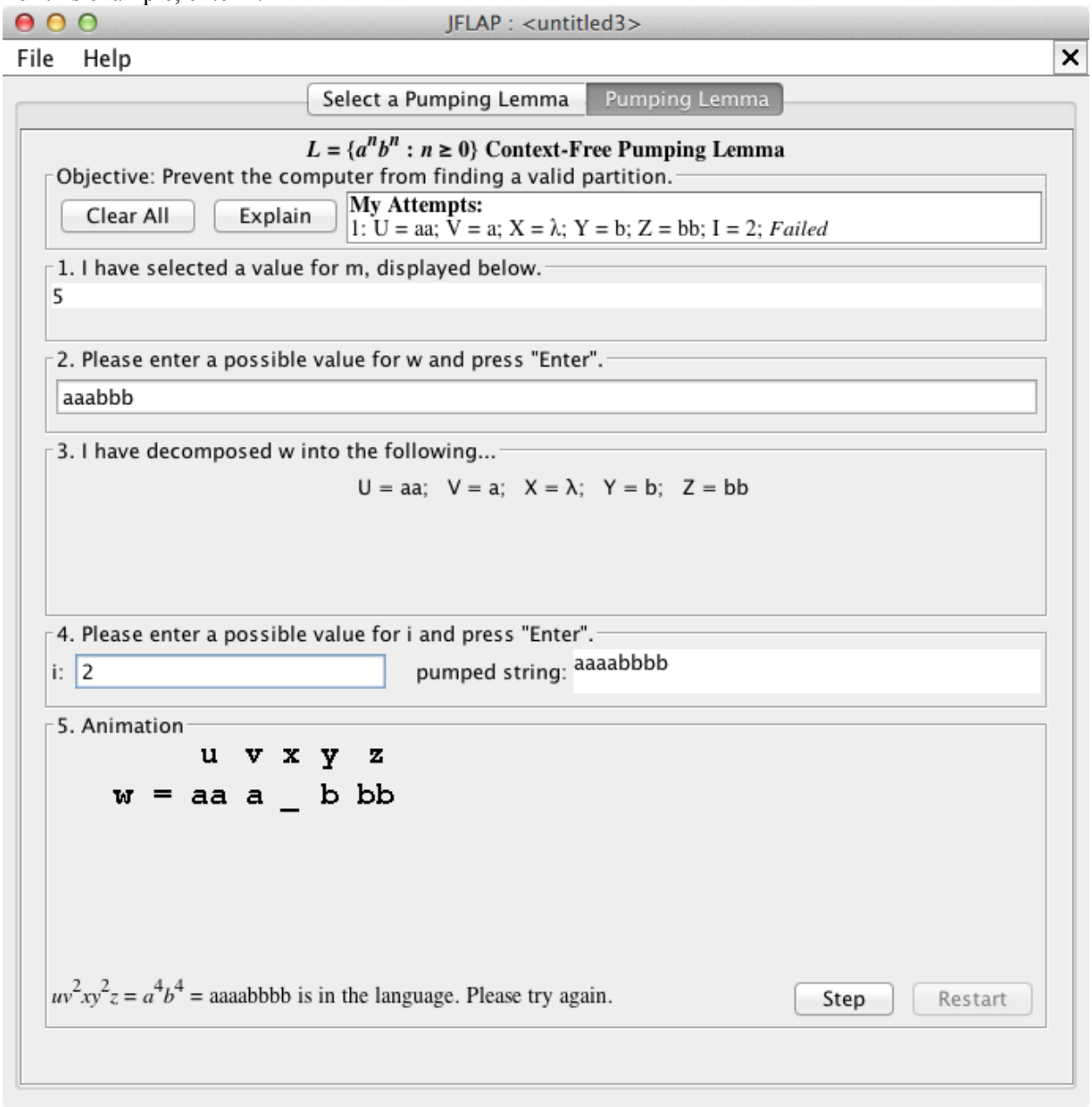
2. Enter a string of length greater than or equal to 5 that is in language L, such as “aaabbb”.



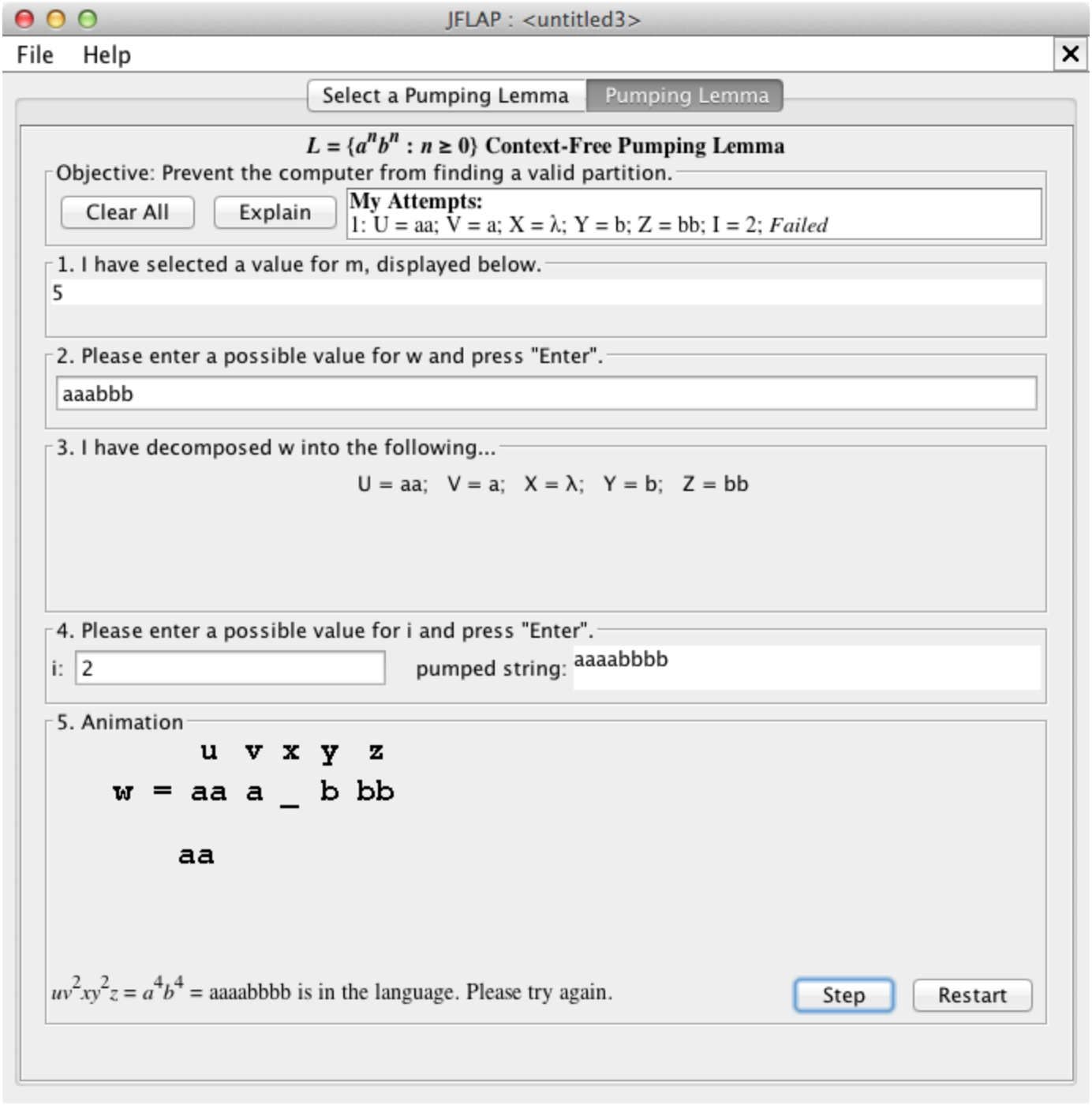
JFLAP prompts for a possible value for i.

3. Consider what value of i ≥ 0, if any, would result in uv^ixy^iz □ L.

For this example, enter 2.



4. Step through the animation to see how the pumping string is created.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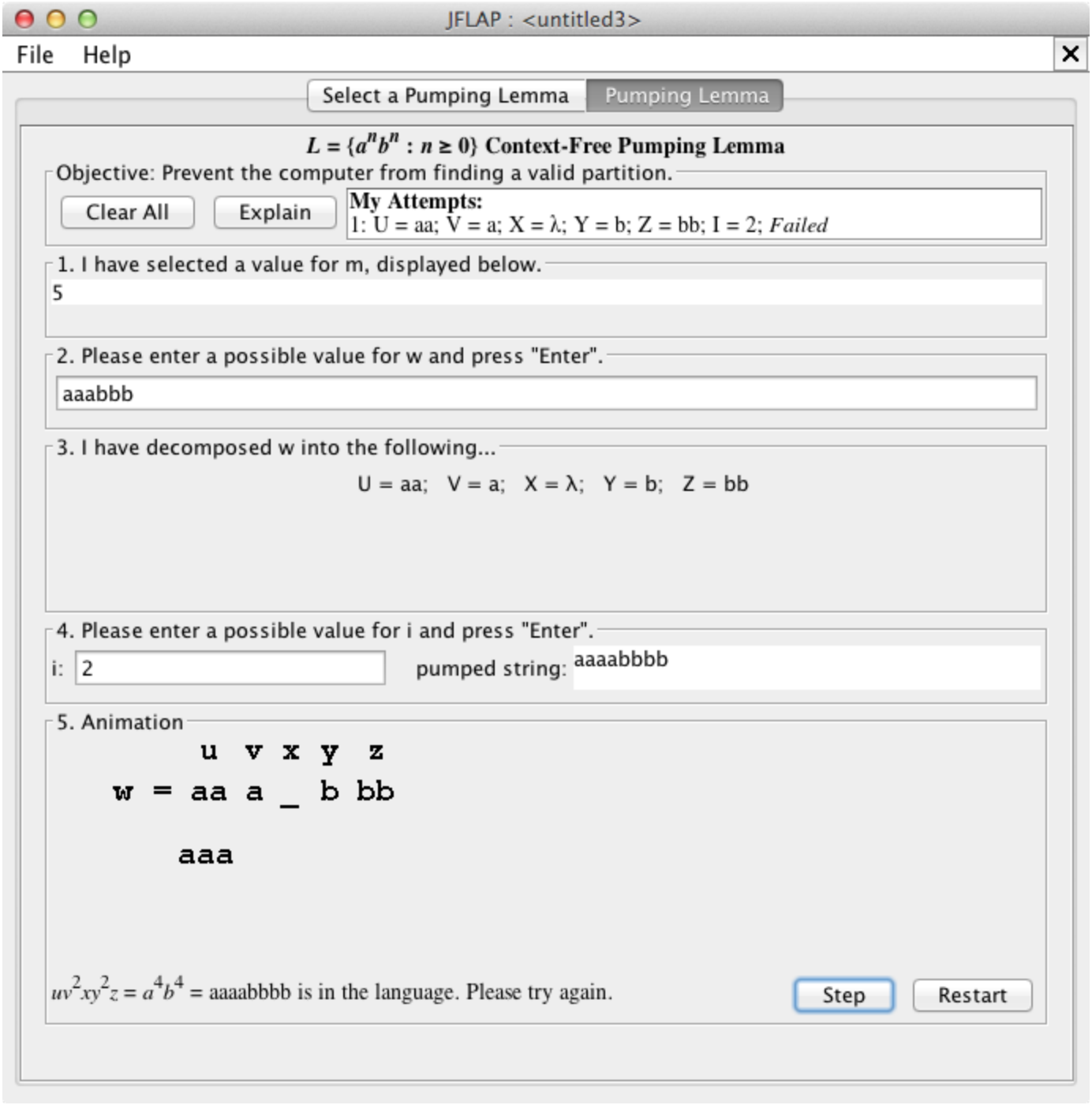
File
Help
Select a Pumping Lemma
Pumping Lemma
$L = \{a^n b^n : n \geq 0\}$ Context-Free Pumping Lemma
Objective: Prevent the computer from finding a valid partition.
Clear All
Explain
My Attempts:
1: U = aa; V = a; X = λ; Y = b; Z = bb; I = 2; Failed
1. I have selected a value for m, displayed below.
5
2. Please enter a possible value for w and press "Enter".
aaabbb
3. I have decomposed w into the following...
U = aa; V = a; X = λ; Y = b; Z = bb
4. Please enter a possible value for i and press "Enter".
i: 2
pumped string: aaaabbbb
5. Animation
u v x y z
w = aa a _ b bb
aa
$uv^2xy^2z = a^4b^4$ = aaaabbbb is in the language. Please try again.
Step
Restart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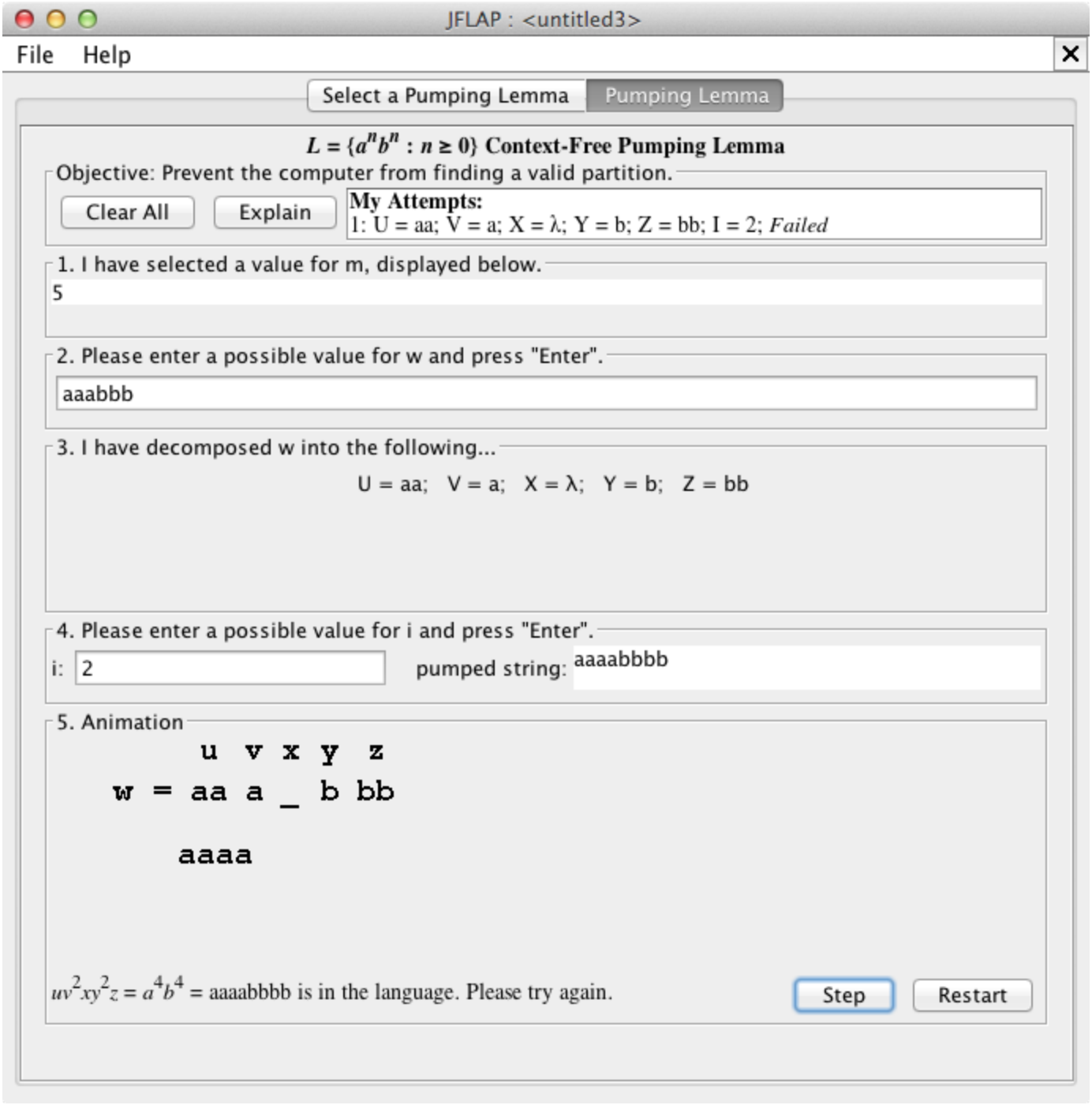
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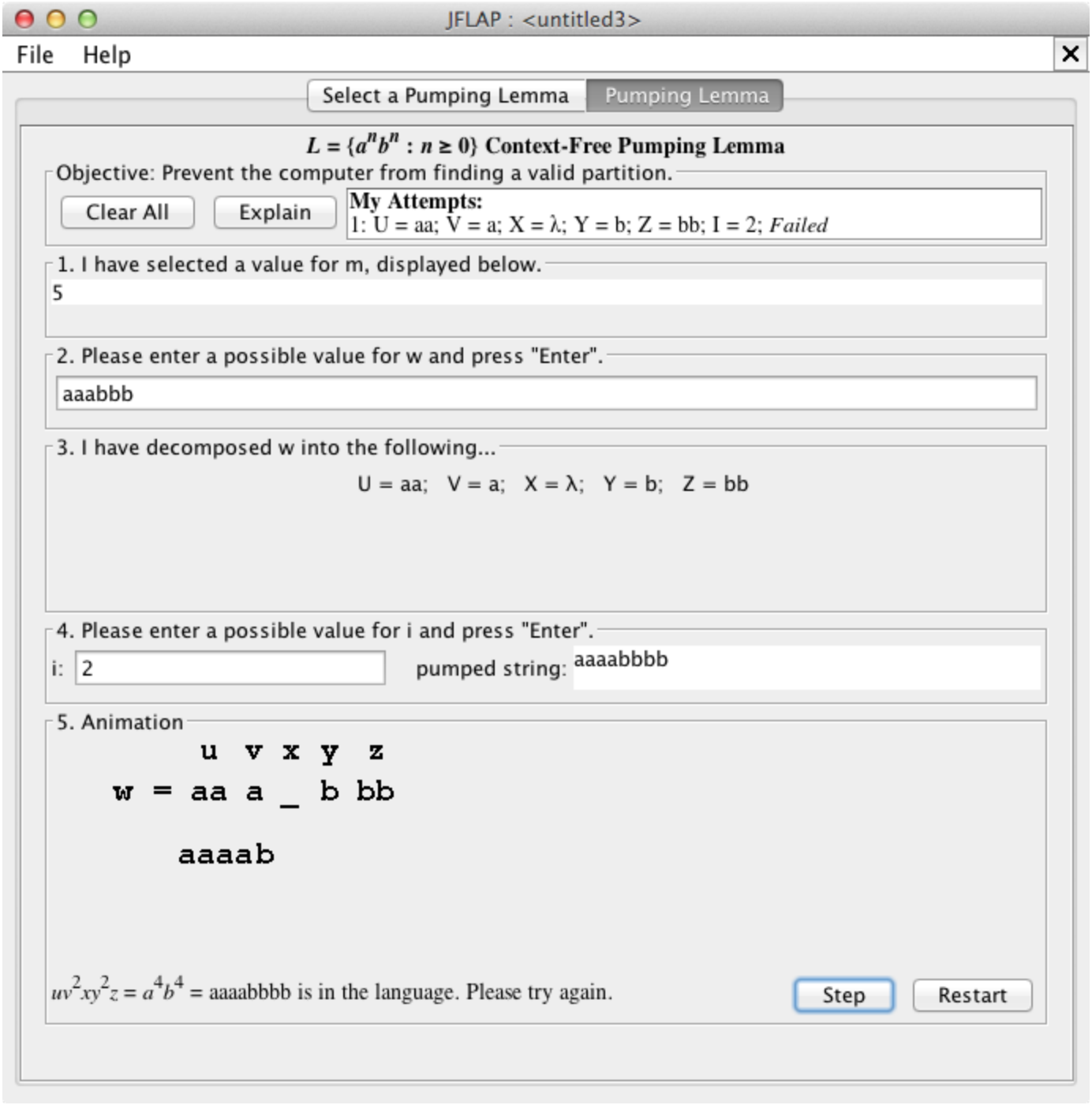
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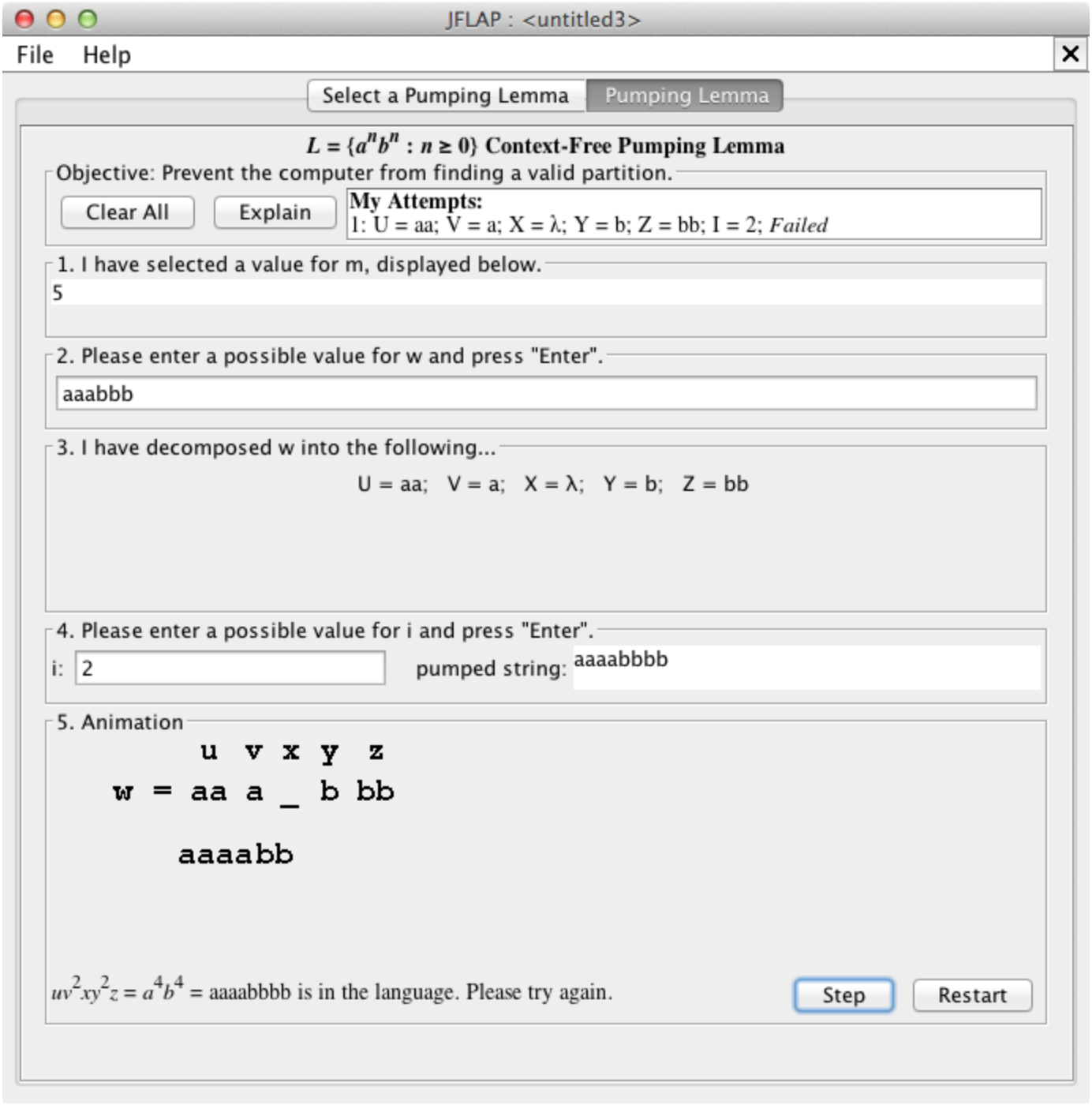
Restart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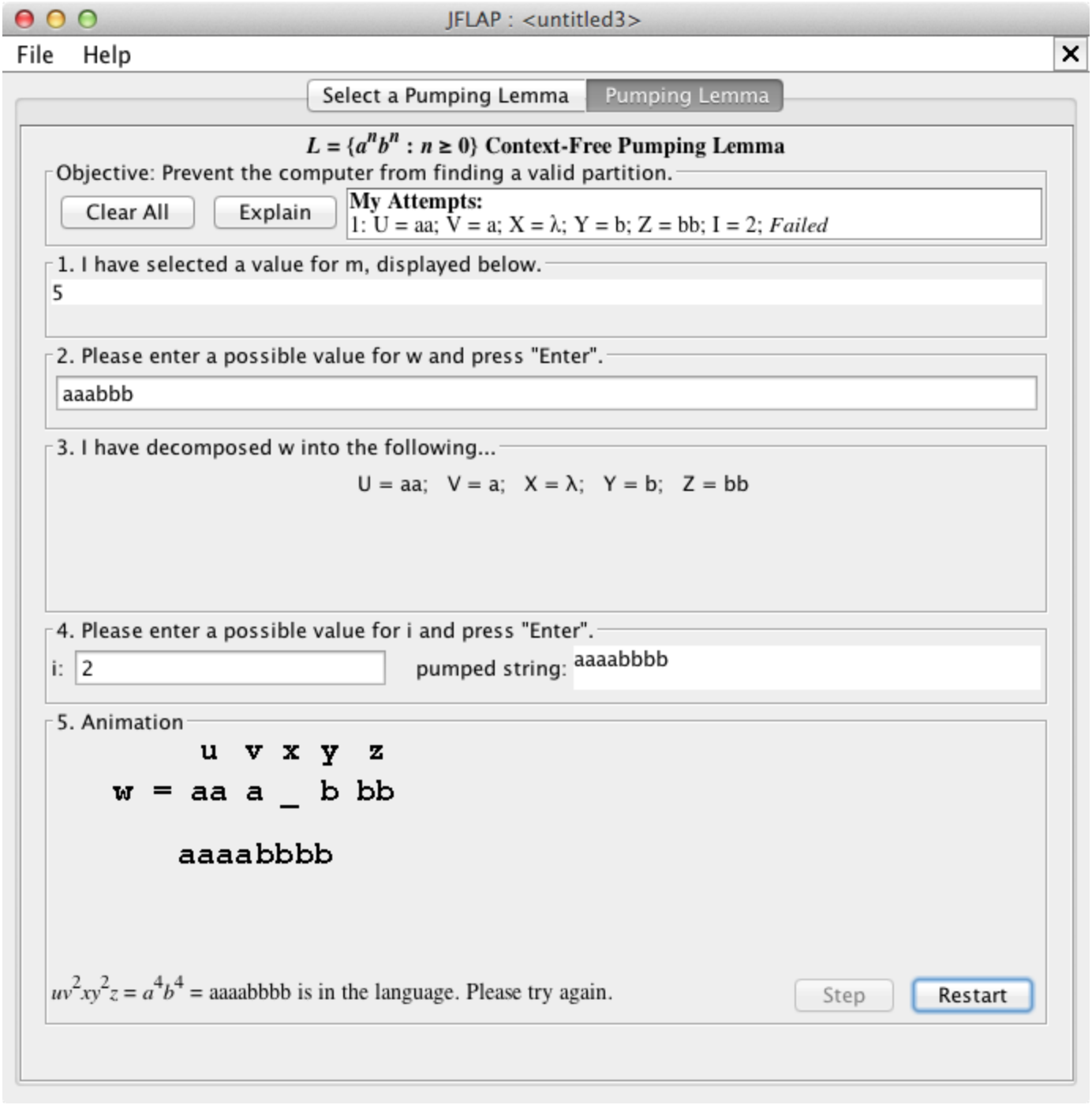
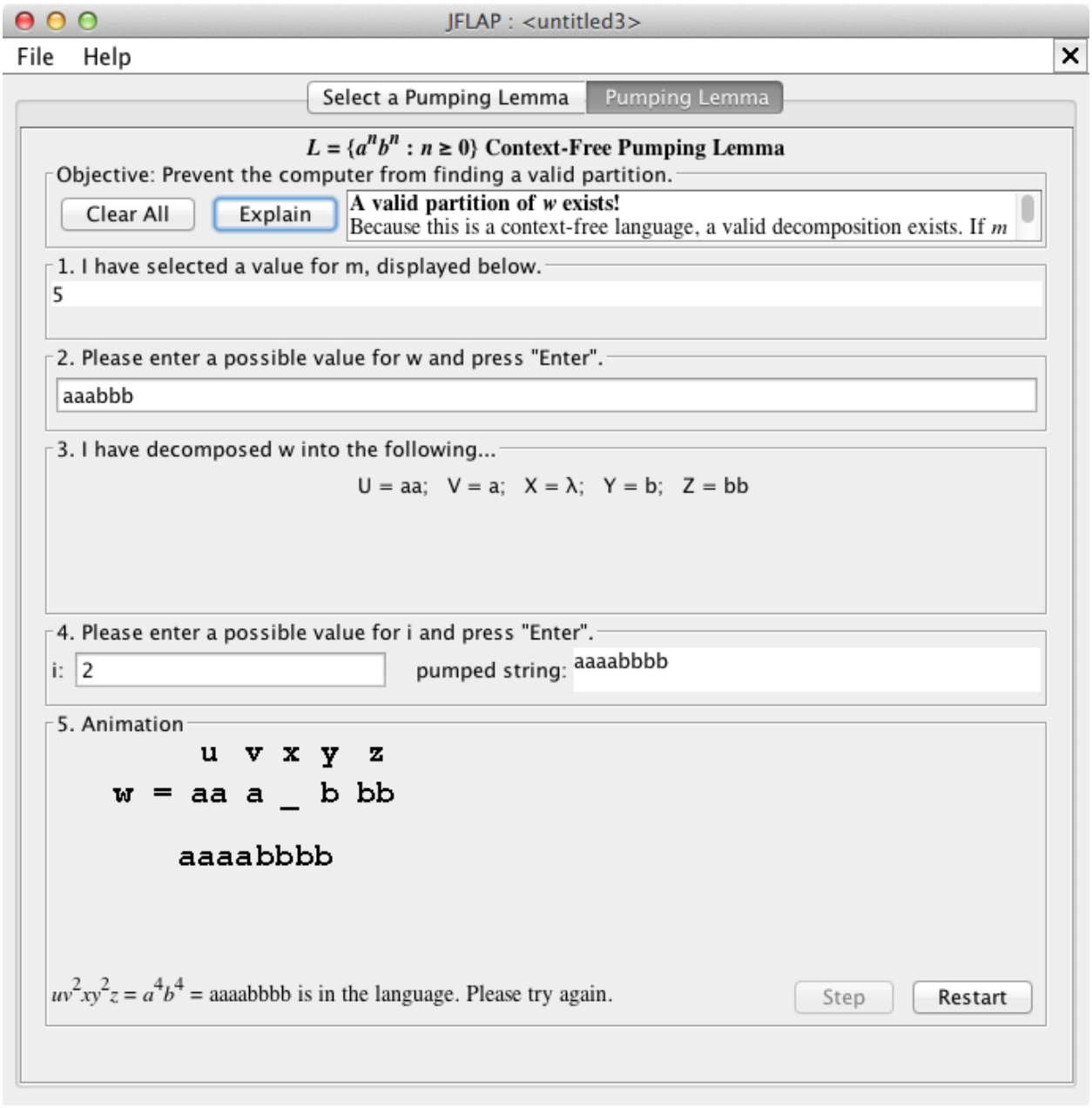
Restart



Because it is not possible to find a string that does not pump within language L for any value of m > 0, it is impossible to prove that L is not a context-free language.

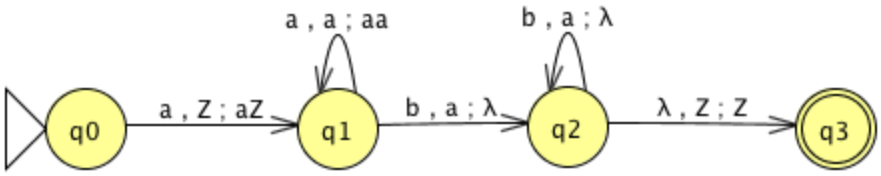
5. Select *Explain* to see the following (detail reproduced below the image due to the need for scrolling).



A valid partition of w exists!
Because this is a context-free language, a valid decomposition exists. If m ≥ 2, one could choose v to be "a" and y to be "b", which will work for all values of i.

My Attempts:
1: U = aa; V = a; X = λ; Y = b; Z = bb; I = 2; Failed

L is in fact a context-free language, as can be demonstrated by defining a Pushdown Automaton that recognizes L, such as the following (see `PDA_anbn.jff`).


a , a ; aa
b , a ; λ
q0
a , Z ; aZ
q1
b , a ; λ
q2
λ , Z ; Z
q3