Example: Context-Free Pumping Lemma_{JP}

Use the JFLAP Context-Free Pumping Lemma game for the lemma $L = \{a^n b^n : n \ge 0\}$

Recall that if L is a context-free language then there exists an integer m > 0 such that any w L with $|w| \ge m$ can be decomposed as the concatenation w = uvxyz, with $|vxy| \le m$, $|vy| \ge 1$, and uv^ixy^iz L for all $i \ge 0$.

Consider what characteristics of a string are necessary for successful decomposition into five concatenated components, the second and fourth of which can be eliminated or repeated.

Walkthrough (see CFPL_anbn.jff)

1. Enter the *Context-Free Pumping Lemma* game, select "Computer goes first", and choose "Select" to the right of "L = { $a^n b^n : n \ge 0$ }".



0 0	JFLAP : <untitled3></untitled3>	
File Help		×
	Select a Pumping Lemma	
First choose who m	akes the first move.	
🔵 You go first 🧕	Computer goes first	
Then select a lemm	a	
	$L = \{a^n b^n c^n : n \ge 0\}$	Select
	$L = \{ww : w \in \{a, b\}^*\}$	Select
	$L = \{a^n b^j a^n b^j : n \ge 0, j \ge 0\}$	Select
	$L = \{ w \in \{a, b, c\}^* : n_a(w) < n_b(w) < n_c(w) \}$	Select
	$L = \{w \in \{a, b, c\}^* : n_a(w) > n_b(w) = n_c(w)\}$	Select
	$L = \{a^i b^j c^k : i > j, i > k\}$	Select
	$L = \{a^n b^n : n \ge 0\}$	Select
	$L = \{a^k b^n c^n d^j : j \neq \mathbf{k}\}$	Select
	$L = \{ww_I w^R : w_I \ge 5, w \& w_I \in \{a, b\}^*\}$	Select
	$L = \{ww_{I}w^{R} : w = w_{I} , w \& w_{I} \in \{a, b\}^{*}\}$	Select
	$L = \{w_I b^n w_2 : n_a(w_I) < n_a(w_2), \mathbf{n_a}(w_I) < n, w_I \& w_2 \in \{a, b\}^*\}$	Select
	$L = \{w_1 c w_2 c w_3 c w_4, : w_1 = w_2 \text{ or } w_3 = w_4, w_i \in \{a, b\}^*, w_i > 0\}$	Select
	$L = \{w_1 v v^R w_2, : n_a(w_1) = n_a(w_2), v > 3, v, w_1, w_2 \in \{a, b\}^*\}$	Select

● ○ ○ JFLAP : <untitled3></untitled3>	
File Help	×
Select a Pumping Lemma Pumping Lemma	
Select a Pumping Lemma Pumping Lemma $L = \{a^n b^n : n \ge 0\}$ Context-Free Pumping Lemma Objective: Prevent the computer from finding a valid partition. Clear All Explain 1. I have selected a value for m, displayed below. 5 2. Please enter a possible value for w and press "Enter".	

In this example, the Computer has selected m = 5.

2. Enter a string of length greater than or equal to 5 that is in language L, such as "aaabbb".

0 0 0	JFLAP : <untitled3></untitled3>	
ile Help		
	Select a Pumping Lemma Pumping Lemma	
	$L = \{a^n b^n : n \ge 0\}$ Context-Free Pumping Lemma	-
Objective: Prevent t	the computer from finding a valid partition.	
Clear All E	Explain	
1. I have selected a	value for m, displayed below.	
5		
2. Please enter a po	ossible value for w and press "Enter".	
aaabbb		
2	and as inter the fallessing	
- 3. I nave decompos	sed w into the following	
	$U = aa; V = a; X = \lambda; Y = b; Z = bb$	
4. Please enter a po	ssible value for i and press "Enter"	
i.		
I.	pumped string:	

JFLAP prompts for a possible value for i.

3. Consider what value of $i \ge 0$, if any, would result in $uv^i xy^i z = L$.

thi	is example, enter	2.	
0	0	JFLAP : <untitled3></untitled3>	
le	Help		
		Select a Pumping Lemma Pumping Lemma	
_		$L = \{a^n b^n : n \ge 0\}$ Context-Free Pumping Lemma	
0	bjective: Prevent	the computer from finding a valid partition.	
	Clear All	Explain My Attempts: 1: U = aa; V = a; X = λ ; Y = b; Z = bb; I = 2; <i>Failed</i>	
1	. I have selected a	a value for m, displayed below.	
5			
	Please enter a p	ossible value for w and press "Enter"	·
	aabbb	ssible value for w and press Enter .	
	aabbb		
4	. Please enter a po	ossible value for i and press "Enter".	
i:	2	pumped string:	
- 5	. Animation		
	u	v x y z	
	w = aa	a_bbb	
uv	$v^2 x y^2 z = a^4 b^4 = aaa$	abbbb is in the language. Please try again.	tep Restart

4. Step through the animation to see how the pumping string is created.

• •	JFLAP : <untitled3></untitled3>	
File	Help	×
	Select a Pumping Lemma Pumping Lemma	
[$L = \{a^{n}b^{n} : n \ge 0\}$ Context-Free Pumping Lemma Objective: Prevent the computer from finding a valid partition. Clear All Explain My Attempts: 1: U = aa; V = a; X = λ ; Y = b; Z = bb; I = 2; Failed	
5	1. I have selected a value for m, displayed below. 5	
	2. Please enter a possible value for w and press "Enter".	
[aaabbb	
	$U = aa; V = a; X = \lambda; Y = b; Z = bb$	
L L	4. Please enter a possible value for i and press "Enter".	
i	i: 2 pumped string: aaaabbbb	
	5. Animation u v x y z w = aa a b bb aa	
u	$av^2xy^2z = a^4b^4$ = aaaabbbb is in the language. Please try again. Step Re	start

• •	JFLAP : <untitled3></untitled3>	
File	Help	×
	Select a Pumping Lemma Pumping Lemma	
[$L = \{a^{n}b^{n} : n \ge 0\}$ Context-Free Pumping Lemma Objective: Prevent the computer from finding a valid partition. Clear All Explain My Attempts: 1: U = aa; V = a; X = λ ; Y = b; Z = bb; I = 2; Failed	
5	1. I have selected a value for m, displayed below.	
	2. Please enter a possible value for w and press "Enter".	
[aaabbb	
	$U = aa; V = a; X = \lambda; Y = b; Z = bb$	
L r'	4. Please enter a possible value for i and press "Enter".	
i	i: 2 pumped string: aaaaDDDD	
	5. Animation uvxyz w=aaa_bbb aaa	
u	$uv^2 xy^2 z = a^4 b^4 = aaaabbbb is in the language. Please try again. Step R$	estart

• •	JFLAP : <untitled3></untitled3>	
File	Help	×
	Select a Pumping Lemma Pumping Lemma	
	$L = \{a^{n}b^{n} : n \ge 0\}$ Context-Free Pumping Lemma Objective: Prevent the computer from finding a valid partition. Clear All Explain My Attempts: 1: U = aa; V = a; X = λ ; Y = b; Z = bb; I = 2; Failed	
5	1. I have selected a value for m, displayed below. 5	
	2. Please enter a possible value for w and press "Enter".	
	aaabbb	
	$U = aa; V = a; X = \lambda; Y = b; Z = bb$	
i:	4. Please enter a possible value for i and press "Enter". i: 2 pumped string: aaaabbbb	
	5 Animation	
	u v x y z w = aa a _ b bb	
	aaaa	
u	$uv^2xy^2z = a^4b^4$ = aaaabbbb is in the language. Please try again. Step Res	tart

0 0	O JFLAP : <untitled3></untitled3>	
File	e Help	>
	Select a Pumping Lemma Pumping Lemma	
	$L = \{a^{n}b^{n} : n \ge 0\}$ Context-Free Pumping Lemma Objective: Prevent the computer from finding a valid partition. Clear All Explain My Attempts: 1: U = aa; V = a; X = λ ; Y = b; Z = bb; I = 2; Failed	
5	1. I have selected a value for m, displayed below.	
	2. Please enter a possible value for w and press "Enter".	
[aaabbb	
	$U = aa; V = a; X = \lambda; Y = b; Z = bb$	
l r'	4. Please enter a possible value for i and press "Enter".	
i	i: 2 pumped string: aaaaDDDD	
	5. Animation u v x y z w = aa a b bb aaaab	
u	$uv^2xy^2z = a^4b^4$ = aaaabbbb is in the language. Please try again. Step Rest	art

0 0	JFLAP : <untitled3></untitled3>	
File	Help	×
	Select a Pumping Lemma Pumping Lemma	
	$L = \{a^{n}b^{n} : n \ge 0\}$ Context-Free Pumping Lemma Objective: Prevent the computer from finding a valid partition. Clear All Explain My Attempts: 1: U = aa; V = a; X = λ ; Y = b; Z = bb; I = 2; Failed	
5	1. I have selected a value for m, displayed below.	
	2. Please enter a possible value for w and press "Enter".	
	aaabbb	
	$U = aa; V = a; X = \lambda; Y = b; Z = bb$	
i	 4. Please enter a possible value for i and press "Enter". 2 pumped string: aaaabbbb 	
	5. Animation u v x y z w = aa a b bb	
u	aaaabb $av^2xy^2z = a^4b^4 = aaaabbbb is in the language. Please try again. Step Re$	start

0	JFLAP : <untitled3></untitled3>	
File	Help	×
	Select a Pumping Lemma Pumping Lemma	
	$L = \{a^{n}b^{n} : n \ge 0\}$ Context-Free Pumping Lemma Debjective: Prevent the computer from finding a valid partition. Clear All Explain My Attempts: 1: U = aa; V = a; X = λ ; Y = b; Z = bb; I = 2; Failed	
- 1 5	. I have selected a value for m, displayed below.	
2	2. Please enter a possible value for w and press "Enter".	
-	aaabbb	
-4	U = aa; V = a; X = λ ; Y = b; Z = bb 4. Please enter a possible value for i and press "Enter".	
i:	2 pumped string: aaaabbbb	
- 5	uvxyz w = aa a _ b bb aaaabbbb	
u	$v^2 xy^2 z = a^4 b^4$ = aaaabbbb is in the language. Please try again. Step	estart

Because it is not possible to find a string that does not pump within language L for any value of m > 0, it is impossible to prove that L is not a context-free language.

5. Select *Explain* to see the following (detail reproduced below the image due to the need for scrolling).

00	JFLAP : <untitled3></untitled3>	
le Help		
	Select a Pumping Lemma Pumping Lemma	
Objective: Prevent	$L = \{a^n b^n : n \ge 0\}$ Context-Free Pumping Lemma the computer from finding a valid partition.	
Clear All	Explain A valid partition of w exists! Because this is a context-free language, a valid decompositi	ion exists. If m
-1. I have selected 5	a value for m, displayed below.	
2. Please enter a p	oossible value for w and press "Enter".	
aaabbb		
- 3 L have decompo	ased winto the following	
5. Thave accompt	M = 22; $M = 2$;	
- 4. Please enter a p	possible value for i and press "Enter".	
i: 2	pumped string: aaaabbbb	
5. Animation		
u	v x y z	
w = aa	a_ b bb	
aaa	abbbb	
$uv^2xy^2z = a^4b^4 = aaa$	aabbbb is in the language. Please try again. Step	Restart
L		

A valid partition of w exists!

Because this is a context-free language, a valid decomposition exists. If $m \ge 2$, one could choose v to be "a" and y to be "b", which will work for all values of i.

My Attempts:

1: U = aa; V = a; $X = \lambda$; Y = b; Z = bb; I = 2; Failed

L is in fact a context-free language, as can be demonstrated by defining a Pushdown Automaton that recognizes L, such as the following (see PDA_anbn.jff).

